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The Systematic Controllability of Cotton Production of Xinjiang based-on Control Theory

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Abstract

This paper is to investigate cotton production systematic controllability. We establish the state equation which is composed gross of cotton manufacture and macroeconomics antecedent index from January 2000 to December 2011. We concluded that cotton production system in Xinjiang is a linear and stationary system and the cotton production system is controllable with macroeconomics stations which influence the rising tendency of production. The level of systematic controllability can be measured by the framework rank of cotton production.

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Keywords: *Controllability; State equation; Xinjiang cotton production system; China*

1. Introduction

Controllability means that it is possible to steer dynamical system from an arbitrary initial state to an arbitrary final state using the set of admissible controls. At present, the theory of deterministic linear control systems is completely mature, which comprises of mathematical model, controllability, observability, stability, optimal control [1]. The earliest research on this topic can be traced back to Kalman's (1962) who introduced discrete-time systems of the form as well as the concept of complete controllability and derived an elegant algebraic test for this property[1-2]. From then on, some other people also come to the same conclusion, for instance, Mohler(1960) initially applies linear control systems in the research of Nuclear reactor[3-6]. Keyou Zhao(1985) examines the reach ability and strong connectivity of the system with control constraints and indicate their relationship and difference[7].

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Linear system controllability researches had made abundant achievements in the 1960s, which was still popular in the next decade. The researches are benefited from the people who have made contributions to the field of study [8-9], which focused on the observability and controllability of linear systems. It is worth mentioning that these concepts are fundamental for understanding if a state observer and a controller are doing well for a hybrid system which can be designed. Controllability and observability have been investigated in paper [10-11] for linear time-varying systems, and in particular for the so-called class of piecewise constant systems (where the matrices in the state-space representation are piece-wise constant functions of time). Although in principle applicable, these results are not enough for one to catch the peculiarities of systems. Based on different arguments, this importance has also been pointed out by Aoki[8], who highlights the equivalence between piece-wise linear systems and interconnections of linear systems and finite automata. Piecewise affine systems are described by the state-space equations.

The linear system controllability is used widely, which represents great academic value and arouses great interest that is mainly motivated by a variety of practical situations. It is not only reflected in the mathematical structure, but also in engineering, economics, biology, ecology and other fields.

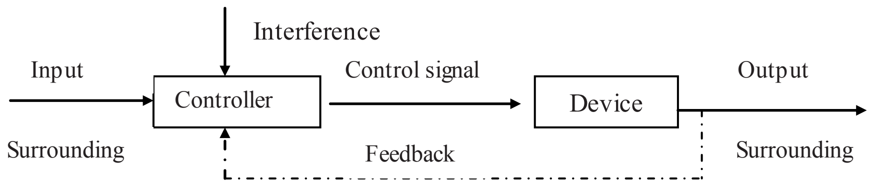
The goal of this paper is to achieve controllability relations between cotton production in Xinjiang and domestic macroeconomics antecedent index, which can not be obtained by the existing results. Besides, we also discuss the macroeconomics antecedent index and cotton manufacture gross fluctuation with controllability of system analysis. The rest of this paper is organized as follows: In Section 2, the relative concept is introduced which includes system, state equitation and Controllability; in Section 3, -the establishment of the model of estimating the effect of China economy antecedent index and cotton production in Xinjiang. Finally, transforming model equation to state equation and analysis of its controllability are discussed in Section 4.

2 Methodology: System, state equitation and Controllability

2.1. System and state equitation

System(S) is a collectivity composed of interconnected all elements with certain function and also a relationship among study object, which is a term of the mathematical control theory. The system is inseparable with another concept which is a state, and the system is also explained as state at a given point, a set of necessary and sufficient information which isolates past behaviour of system from future behaviour. It is the minimum of past and present information about system. There is another inseparable concept with system, which is named as state. The state is a generalization of the condition that is in a particular moment of system. And also state is a set of necessary and sufficient information which is separated past behaviour of system from future behaviour, which is the minimum of past and now information about system. So it is introduced that state variable definition is to describe systemstate.

In research of control theory, we only consider control system. The control system refers to a random system which adjusts and controls information flow, energy flow, material flow according to a kind of mode. The control system is consisted as two basic parts with main control system (controller) and the controlled system (device), as shown in chat 1.



Chat.1. System structure

Firstly, the system state is given the definition that the spaces are all the n-dimensional Euclidean space R^n

Definition 1: The system state is a minimal aggregate with a set of variables, which contains enough information in past system to calculate future state so that these variables are referred to state variables, donate as $x_1, \cdots, x_n$; $n \times 1$ Column vector is called as state vector $x = (x_1, \cdots, x_n)^T$.

Contact system (S) is a set of input variables ($u_1, \cdots, u_p$) and a set of output variables ($y_1, \cdots, y_m$). For the sake of convenience, the input variables $u_1, \cdots, u_p$ is represented by $p \times 1$ the column vector $u = (u_1, \cdots, u_p)^T$, the output vector(y) is defined as $m \times 1$ the column vector $y = (y_1, \cdots, y_m)^T$.Assuming that the input vector (u) and output vectors (y) are all function of time-output vector (t), they can be donate as $u(t)$ and $y(t)$.

Definition 2: Aggregation which is input vector (u) at a time (t) with all possible is called input space; Aggregation which is output vector(y) at a time (t) with all possible is called output space; Aggregation which is state vector at time with all possible is called a state space

According to definition 1, the state vector (x) can be regarded as:

$$x(t) = F(x(t_0), u(t)) \tag{1}$$

F is Single value function of independent variable. It is indicated from form (1) that future state of system(S) can be identified only with a kind of $u(t)$ where all of $x(t_0)$ in state space and $u(t)$ in input space. And y(t) can be described as follow:

$$y(t) = G(x(t_0), u(t)) \tag{2}$$

G is Single value function of change independent variable.

Definition 3: form (1) and form (2) are called state equation of system(S)

This paper is to focus on study discrete-time system which the input and the output is linked by using of differential equations, and it is a linear and stationary system, sing input and sing output system, and form is as follow

$$\begin{cases} x(k+1) = Ax(k) + Bu(k) \\ y(k) = Cx(k) + Du(k) \end{cases} \tag{3}$$

In formula (3), it shown that A is $n \times n$ Constant matrix, B is $n \times 1$ Constant column vector, C is $1 \times n$ Constant row vector and D is Constant.

2.2. Introduction of establishing state equation method

This part is to introduce method which is establishing state equation on line dynamic system configuration state vector, and the form of constant differential equations can be describe as flow:

$$\begin{aligned} & y(k+n) + a_1 y(k+n-1) + \cdots + a_{n-1} y(k+1) + a_n y(k) \\ & = b_0 u(k+n) + b_1 u(k+n-1) + \cdots + b_n u(k) \\ & (k = 0,1,2,\cdots) \end{aligned} \tag{4}$$

Assuming that state vector as below

$$x_1(k) = y(k) - h_0 u(k)$$

$$x_2(k) = x_1(k+1) - h_1 u(k)$$

$$\cdots,\cdots$$

$$x_n(k) = x_{n-1}(k+1) - h_{n-1} u(k)$$

From the above, $h_0, h_1, \cdots, h_{n-1}$ and h_n can be identified by follow equations

$$h_0 = b_0$$

$$h_1 = b_1 - a_1 b_0$$

$$h_2 = b_2 - a_1 h_1 - a_2 h_0 \cdots, \cdots$$

$$h_n = b_n - a_1 h_{n-1} - \cdots - a_{n-1} h_1 - a_n h_0$$

It can be received formula (4) matching state equation (3)

$$A = \begin{pmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ -a_n & -a_{n-1} & -a_{n-2} & \cdots & -a_1 \end{pmatrix}, \quad B = \begin{pmatrix} h_1 \\ h_2 \\ \vdots \\ h_n \end{pmatrix} \tag{5}$$

$$C = (1, 0, \cdots, 0), \quad D = b_0$$

2.3. Knowledge of controllability

The system controllability is an ability which means the influence of input variables on the state variables. Deciding the general control system controllability is extremely difficult with profound mathematical theory. But linear system controllability has formed a relatively complete theory in 1960's. According to formula (3), it can be defined as controllability as follows:

Definition 4: In the section of $[k_0, k_0 + l]$, input variable can be selected $u(k)$ ($k_0 \le k \le k_0 + l$) for any random initial state x_0, it can make match state vector $x(k)$ from $x(k_0) = x_0$ to $x(k_0 + l) = 0$. So system (3) can be controlled in section of $[k_0, k_0 + l]$. If all kinds of $k > k_0$ systems are controlled, the system is completely controlled.

To system (3), the complete controllability qualification is as below theorem 1, and the detailed proof may refer to "Mathematical Control and Applications" (Zongqi Deng, 1997)

Theorem 1: The criterion is that the order of controllability matrix equals to the number of rows or columns. System (3) complete control condition is $rank(Q_n) = n$, $Q_n = (B, AB, \cdots, A^{n-1}B)$

3 Empirical results and analysis

3.1. Mode of establishment

System's mathematical model should be established and transformed into state equation before analyzing controllability of cotton production in Xinjiang.

In theory, demand function of cotton product in Xinjiang is mainly affected by the domestic market, the level of income and relative price.

$$y_{cotton} = f(i, p)$$

Macroeconomics antecedent index can reflect tend towards of domestic economic development, which is mainly composed of four index, four index refer to means of production price in total index which is a converse target, finance payout, industrial enterprises deficit and macroeconomics boom index, all of the data are covered during the period of January 2000 to December 2011. In equation, if cotton income level increases, it will promote the yield of cotton growth. If relative cotton price rises, it will reduce the Chinese cotton imports,.

Therefore, cotton production will be greatly increasing. By anglicizing of cotton consumption structure in Xinjiang, cotton intermediate goods and capital goods are regarded as a very large proportion, and the proportion of little raw cotton. Thus, it should be attempted to use the macroeconomics antecedent index (L) instead of income variable, it will be much better to put emphasis on the study of cotton production system. At the same time, in the short-term analysis, assuming that the cotton price is maintained at a certain level of stability, and only China economic development is main factor affecting cotton production. Based on these analyses, it can be converted to the following equation:

$$y_{\mathrm{cot}ton} = f(L) \tag{7}$$

The equation(7) is attempted to make use of autoregressive distributed lag model (ADL) to do the estimation, and the data are covered during the period of January 2000 to December 2011. The explained factor is monthly gross growth rate cycle fluctuations of cotton manufacture. ($y_{\mathrm{cot}ton}$) The explanatory factors (L) include macroeconomics antecedent index from the database Wind Information Co., Ltd (Wind Info).

However, the results of LM test with the preliminary ADL model received residual series are autocorrelation. Therefore, relevant measures are taken to eliminate, introduced $ar(1)$ to eliminate correlation. The final model estimation results are as follows (in brackets is the standard error):

$$y_{\mathrm{cot}ton} = 2.247 y_{\mathrm{cot}tom}(-1) - 1.992 y_{\mathrm{cot}ton}(-2) + 0.672 y_{\mathrm{cot}ton}(-3) + 0.062 L(-8) + 1.08 ar(1) \tag{8}$$

(0.02) (0.15) (0.01) (0.05) (0.07)

In the mode, adjusted R^2 is 0.82 and probability is 0.34 with LM test, which does not refuse the hypothesis that residual series which is not resist autocorrelation.

3.2. Analysis of controllability

Based on the model(8) estimation results, by ignoring $ar(1)$ in the model, we consider only the major part as systemmodel equations and analysis system only with two variables what cotton manufacture gross and macroeconomics antecedent index is formed, denoted as S2, and then transformed into state equation form. Finally, the analysis of the cotton production system controllability is illustrated. Differential equation can be denoted as follows:

$$y_{\mathrm{cot}ton} = 2.247 y_{\mathrm{cot}tom}(-1) - 1.992 y_{\mathrm{cot}ton}(-2) + 0.672 y_{\mathrm{cot}ton}(-3) + 0.062 L(-8) \tag{9}$$

Then we can use method (1) to change state equation (9)

$$\begin{cases} x(k+1) = Ax(k) + Bu(k) \\ y(k) = Cx(k) + Du(k) \end{cases}$$

$$x = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_8 \end{pmatrix}, \quad A = \begin{pmatrix} 0 & 1 & 0 & 0\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0\ 0 & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots\cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0\ 0 & 0.672 & -1.992 & 2.247 \end{pmatrix}, \quad B = \begin{pmatrix} 0 \\ \cdots \\ 0 \\ 0.062 \end{pmatrix}$$

$$C = (1, 0, \cdots, 0), \quad D = 0$$

$$rank(Q_8) = rank(B, AB, A^2B, \cdots, A^7B) = 8$$

According to theorem 1, system (S_2) can be controlled. It indicates that domestic economic development environment can affect the growth of cotton production of fluctuations, and the composition of the simple system is controllable from the historical data in the estimation of system equation shows. Therefore, the domestic economic development environment, such as demand, investment growth of import promotion process can be controlled. By taking the measures that vigorously expand domestic demand and promote consumption of macroscopical adjusting, there will be an effective growth for imports, which promotes trade balance development.

4. Conclusions

In this paper, we have discussed issue of systematic controllability of cotton production in Xinjiang. The empirical results show that this systematic controllability method deals well with the problem and it will be transformed to state equation after the establishment of mathematics mode of cotton manufacture gross and macroeconomics antecedent index. The results show that the cotton product system is controllable with macroeconomics station, which influences the rising tendency of production. However, the percentage of textiles and clothing consumption demands, which mainly to intermediate production and cotton spinning should be expanded greatly under the circumstances of the kind of cotton goods. In order to finish the restructuring task of the cotton spinning sector, the textile industry will have to initiate the restructuring of woolen and silk sectors.

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